Optimal Policy Without Rational Expectations: A Sufficient Statistic Solution

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    3. Solve the optimal policy problem
- Lack of generality is a problem: no consensus on how expectations are formed (beyond FIRE fails), precisely how they affect the real economy, etc.

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  - Measure peoples' expectations
  - Know how decisions (equilibrium conditions) are directly distorted by non-rational expectations

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  - But the **belief distortion** is still a sufficient statistic for the optimal policy!
- Work through simple examples for both cases

General Framework

- General model:

$$B_{X1}\mathbb{E}_t^b\left[X_{t+1}\right] = B_{X0}X_t + B_Y Y_t + B_G G_t \tag{1}$$

  - $\mathbb{E}_t^b\left[\cdot\right]$: behavioral expectation of type b
  - X_t: endogenous variables
  - Y_t: exogenous variables
  - G_t: policy variables
- A **behavioral expectations equilibrium**:
  1. X_t, Y_t, and G_t satisfy the equilibrium condition (1)
  2. Y_t, X_t and G_t are stationary, linear in the history of shocks $\{\omega_{t-j}\}_{j=0}^{\infty}$
  3. G_t satisfies a policy rule
- For now: assume FIRE equilibrium X_t^* is welfare-maximizing, unique

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- See Adams (2023) for technical details

Belief Distortions

- $\mathbb{E}_t[\cdot]$ (with no b specified) denotes the *rational expectation*
- Define the **belief distortion** as

$$\mathbb{D}_t^b[X_{t+1}] \equiv \mathbb{E}_t^b[X_{t+1}] - \mathbb{E}_t[X_{t+1}]$$

- In a model, it is specific to the type b of behavioral expectations
- In the data, requires measuring agents' expectations $\mathbb{E}_t^b[X_{t+1}]$, and estimating the rational expectation $\mathbb{E}_t[X_{t+1}]$

What Must Optimal Policy Do?

Lemma

If there is a time series of policy instruments G_t such that the non-rational equilibrium is consistent with the policy-less FIRE equilibrium, then G_t satisfies

$$B_{X1}\mathbb{D}_t^b[X_{t+1}] = B_G G_t$$

Proof Outline:

- In the FIRE equilibrium with $G_t = 0$, endogenous vector X_t^* satisfies:

$$B_{X1}\mathbb{E}_t\left[X_{t+1}^*\right] = B_{X0}X_t^* + B_Y Y_t$$

- Subtract from the non-rational model to get:

$$B_{X1}\mathbb{E}_t^b\left[X_{t+1}\right] - B_{X1}\mathbb{E}_t\left[X_{t+1}^*\right] = B_{X0}(X_t - X_t^*) + B_G G_t$$

- Impose $X_t = X_t^*$, and rearrange.

Sentiment Spanning: Definition

- What policy instruments are enough to recover FIRE?
- Some notation:
  - B_{C1} is submatrix of B_{X1} corresponding to control variables (there is no belief distortion about pre-determined state variables)
  - $P_G \equiv B_G(B_G' B_G)^{-1} B_G'$ is projection onto column space of B_G.

Condition (Sentiment Spanning)

The macroeconomic model defined in (1) *is said to satisfy Sentiment Spanning if*

$$(I - P_G)\, B_{C1} = 0$$

Sentiment Spanning in Practice

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- Examples:

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- Policymaker does not need to know the whole model to evaluate SS! Needs to know:
  - How expectations affect decisions (B_{C1})
  - How policy instruments distort economy (B_G)

Optimal Policy: The Sufficient Statistic

Theorem
If a model satisfies Sentiment Spanning, then the policy rule

$$G_t^{\dagger} = (B_G' B_G)^{-1} B_G' B_{C1} \mathbb{D}_t^b [X_{t+1}^C] \tag{2}$$

recovers the FIRE equilibrium.

- The belief distortion $\mathbb{D}_t^b [X_{t+1}^C]$ is a sufficient statistic!
- Why does Sentiment Spanning matter? Invert the Lemma $B_{X1} \mathbb{D}_t^b [X_{t+1}] = B_G G_t$

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- Optimal policy: tax capital when agents are overly optimistic about future returns

Example 1: Decentralized Equilibrium Conditions

- Policymakers have light information requirements:

Euler Equation: $\tau_t = \sigma c_t + \mathbb{E}_t^b[-\sigma c_{t+1} + \overline{R} r_{t+1}]$

Labor Supply: $w_t = \sigma c_t + \eta n_t$

Production Function: $y_t = a_t + \alpha k_{t-1} + (1-\alpha) n_t$

Capital Demand: $r_t = y_t - k_{t-1}$

Labor Demand: $w_t = y_t - n_t$

Resource Constraint: $\overline{Y} y_t = \overline{C} c_t + \overline{K}\left(k_t - (1-\delta) k_{t-1}\right)$

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- Optimal policy: $\tau_t^\dagger = \mathbb{D}_t^b[-\sigma c_{t+1} + \overline{R} r_{t+1}]$

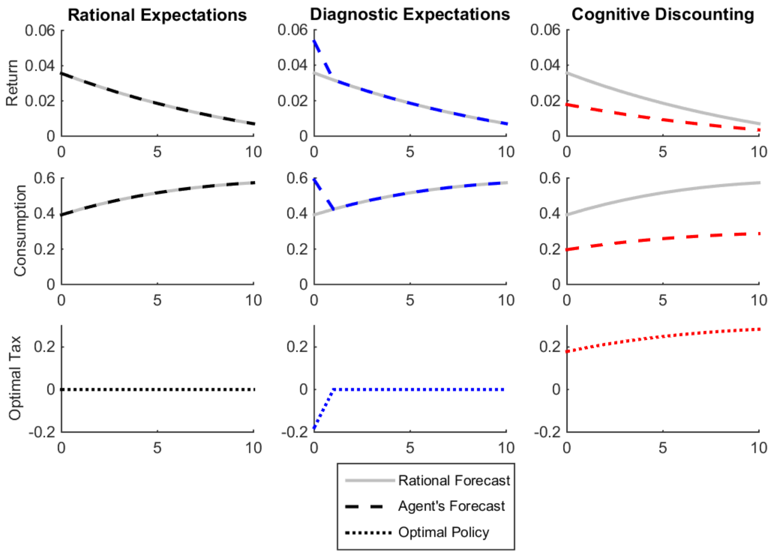
Example 1: Types of Behavioral Expectations

Rational Expectations: $\mathbb{E}_t^{RE}[x_{t+1}] = \mathbb{E}_t[x_{t+1}]$

Diagnostic Expectations: $\mathbb{E}_t^{DE}[x_{t+1}] = (1+\theta^{DE})\mathbb{E}_t[x_{t+1}] - \theta^{DE}\mathbb{E}_{t-1}[x_{t+1}]$

Cognitive Discounting: $\mathbb{E}_t^{CD}[x_{t+1}] = \theta^{CD}\mathbb{E}_t[x_{t+1}]$

Example 1: Response of Expectations to a Productivity Shock



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- Both cases: raise interest rates when agents misperceive the economy to be running hot

Example 2: Optimal Monetary and Fiscal Policy in BNK

New Keynesian Phillips Curve: $\psi f_t = \kappa y_t - \pi_t - z_t^{PC} + \beta \mathbb{E}_t^b[\pi_{t+1}]$

Euler Equation: $i_t = -\sigma y_t - z_t^{EE} + \mathbb{E}_t^b[\sigma y_{t+1} + \pi_{t+1}]$

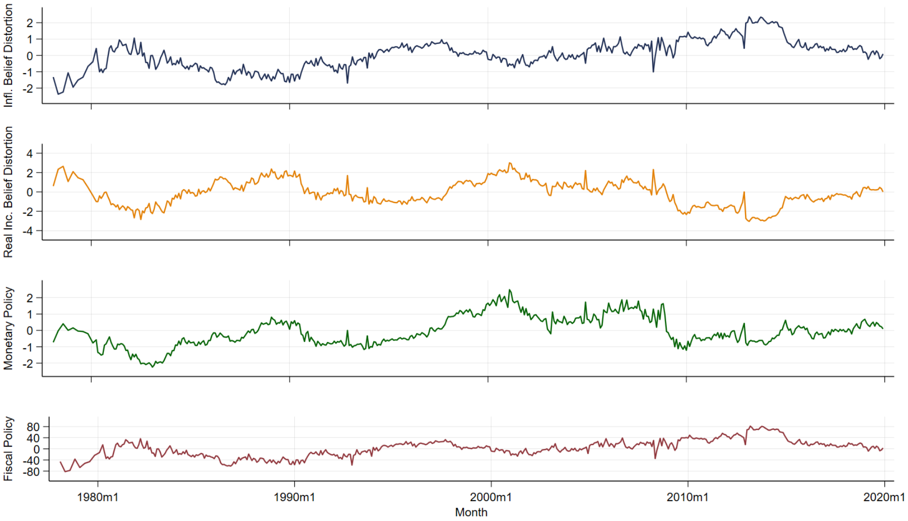
Expectation components of optimal policy are:

$$\hat{f}_t^\dagger = \frac{\beta}{\psi} \mathbb{D}_t^b \left[\hat{\pi}_{t+1}\right] \qquad \hat{i}_t^\dagger = \mathbb{D}_t^b \left[\sigma y_{t+1} + \pi_{t+1}\right]$$

Implementation:

- Measure agents' expectations $\mathbb{E}_t^b[\cdot]$,
- Estimate the rational expectation, e.g. with a VAR (Adams and Barrett 2024)

Example 2: Estimated Belief Distortions and Implied Policies



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- ... intuition goes through, although implementation may change

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- Policymakers now need to know the whole *economic* model
- … but they still do not need to know how expectations are formed!

Welfare Objective

- First-best equilibrium: X_t^*, with FIRE-optimal policy G_t^*
- Policymakers with no information commit to a policy rule (Rottemburg and Woodford 1997)
- Minimize quadratic loss for some W:

$$\min \mathbb{E}\left[(X_t - X_t^*)' W (X_t - X_t^*)\right]$$

Optimal Policy Without Sentiment Spanning

Theorem
The constrained-optimal policy rule is

$$G_t^{\dagger} = \underbrace{B_G^{+} P_W B_{C1} \mathbb{D}_t^{b}[X_{t+1}^{C}]}_{\textit{expectation component}} + G_t^{RE}$$

▸ *matrix details*

- Economic component G_t^{RE} follows the *FIRE policy rule*

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- Expectation component $B_G^{+} P_W B_{C1} \mathbb{D}_t^b [X_{t+1}^C]$ has the *same information requirements* as the original case with sentiment spanning
- $\Longrightarrow$ if you already have the FIRE optimal policy, adding the response to non-rational expectations requires no additional modeling assumptions, only measuring the belief distortion!

Example 3: Optimal Monetary Policy Alone in BNK

New Keynesian Phillips Curve: $0 = \kappa y_t - \pi_t - z_t^{PC} + \beta \mathbb{E}_t^k[\pi_{t+1}]$

Euler Equation: $i_t = -\sigma y_t - z_t^{EE} + \mathbb{E}_t^b[\sigma y_{t+1} + \pi_{t+1}]$

Expectation component of optimal policy is:

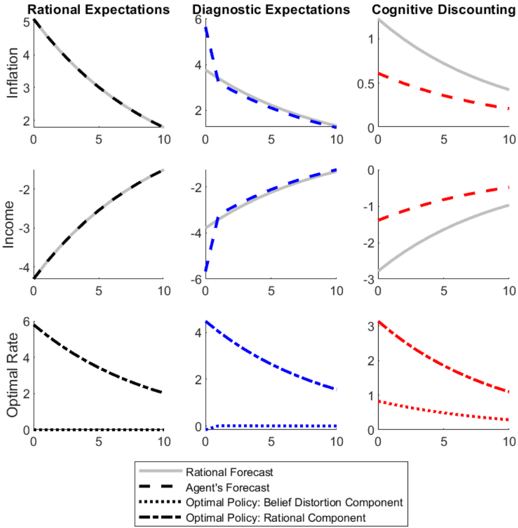
$$\hat{i}_t^{\dagger} - i_t^{RE} = \sigma \mathbb{D}_t^b[y_{t+1}] + \left(1 - \beta \frac{b_\pi \kappa \sigma}{b_\pi \kappa^2 + b_y}\right) \mathbb{D}_t^b[\pi_{t+1}]$$

which cannot recover FIRE without an additional tool.

If $\left(1 - \beta \frac{b_\pi \kappa \sigma}{b_\pi \kappa^2 + b_y}\right) > 0$, raise rates when agents misperceive economy is "running hot".

▸ Speedy Conclusion

Example 3: Response of Expectations to a Cost-Push Shock



What if Belief Distortions are Measured with Error?

- Policymaker's observation D_t of the belief distortion is

$$D_t = \xi \mathbb{D}_t^b [X_{t+1}] + \upsilon_t$$

with i.i.d. measurement error $\upsilon_t \sim N(0, \sigma_\upsilon^2)$

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- Form the *policymaker's* nowcast of the belief distortion $\mathbb{D}_t^b[X_{t+1}^C]$ conditional on info. set Ω_t (D_t and other observables):

$$\hat{D}_t = \mathbb{E}[\mathbb{D}_t^b[X_{t+1}^C] | \Omega_t]$$

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- **Theorem** *The constrained-optimal policy rule is*

$$G_t^\dagger = B_G^+ P_W B_{C1} \hat{D}_t + G_t^{RE}$$

... same as the solution without Sentiment Spanning, except using $\hat{D}_t$!

What if Expectation Formation is Endogenous? (Framework)

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Optimal Policy Without Sentiment Spanning

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The constrained-optimal policy rule is

$$G_t^{\dagger} = B_G^{+} P_W B_{C1} \mathbb{D}_t^{b}[X_{t+1}^{C}] + B_G^{+} P_W B_{X1} \mathbb{E}_t[X_{t+1} - X_{t+1}^{*}] + G_t^{*}$$

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  2. Economic component: optimal policy for FIRE model

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3. What if expectation formation is endogenous?
    - Optimal rule unchanged; lose existence/uniqueness from the main theorem

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